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Numerical simulation of glacial-valley cross-section evolution

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Abstract

A numerical model for glacial-valley cross-section evolution has been developed. The model allows the simulation of the development of U-shaped valleys by coupling an ice flow model in a transverse section with an erosion model. The core of the cross-section development model is the calculation of the two-dimensional flow speed field in a transverse cross-section considering the lateral drag from glacier side walls and the basal-stress dependent sliding speed. Assuming that the glacial erosion is a quadratic function of the sliding speed, the model shows rapid evolution of a V-shaped profile into a recognizable glacial form with sliding velocities ranging from 3 m a^{-1} to 8 m a^{-1}.

1. Introduction

The U-shaped valley is one of the most well-known products of alpine glaciation, yet relatively little is known how ice flow and glacial erosion interact in the development of this characteristic form and why glacial erosion should produce such a form. Empirical studies have confirmed the general notion that many glaciated valleys have approximately parabolic (U-shaped) cross sections (Graf, 1970; Doornkamp and King, 1971; Girard, 1976; Aniya and Welch, 1981). To investigate the formation processes of U-shaped valleys, ice flow at the glacier bed was studied (Johnson, 1970) and quantitative abrasion and plucking models depending on ice velocity and effective pressure have been proposed (Boulton, 1974; Hallet, 1979; Roberts and Rood, 1984). Moreover, coupling of ice flow and erosion models allowed better understanding of the evolution of cross-sectional (Harbor, 1990, 1992, 1995) and longitudinal profiles (MacGregor *et al.*, 2000).

With his glacial cross-section evolution model and by assuming a quadratic function of the sliding velocity for an erosion law, Harbor successfully simulated a proper erosion pattern (central minimum in the basal sliding velocity at the valley center) for the U-shaped channel development. However, Harbor provided little information on the computation of the basal shear stress, which is required for successful erosion modeling. Particularly the stress conditions required for the development of a U-shaped valley have not been described.

This paper presents the details of two-dimensional flow pattern computation and its coupling with the subglacial erosion model. The developed valley evolution model was tested by investigating the influence of the lateral shear stress component on the formation of a U-shaped valley.

2. Methods

2.1. *Flow model*

The problem to be solved is the velocity field in a transverse cross-section of a glacier with a uniform geometry along the glacier. The Cartesian coordinate system is taken with the x-axis along the glacier, y across the glacier, and z perpendicular to the x-y plane (Fig. 1). Momentum balance of the ice in the x-direction is

$$\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}=\rho g\frac{\partial S}{\partial x}, \tag{1}$$

where τ_{xy} and τ_{xz} are the shear stresses, ρ is the density of ice, g is the acceleration of gravity, and S is the surface elevation. Glen's flow law is used as a constitutive relation, so that

$$\frac{\partial u}{\partial y}=2F\tau_{xy}, \quad \frac{\partial u}{\partial z}=2F\tau_{xz}. \tag{2}$$

The term F is the fluidity, defined as

$$F=A\left(\tau_e^2+\tau_0^2\right)^{\frac{n-1}{2}}. \tag{3}$$

Here, τ_e is the effective stress, and the rate factor A and the flow-law exponent n are material parameters. We used the common values of $n=3$ and $A=214\ \mathrm{MPa}^{-3}\ \mathrm{a}^{-1}$

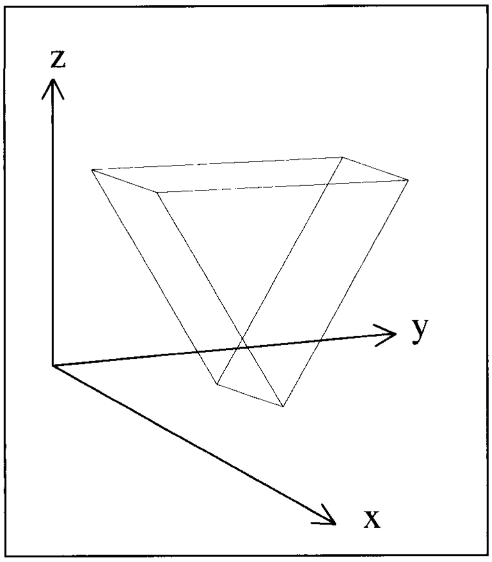

Fig. 1. Initial glacier shape and the coordinate system used in the simulation.

(Paterson, 1994). τ_0 was introduced to avoid the mathematical singularity caused by an infinite viscosity when stresses approach zero (Blatter, 1995) and a value of $\tau_0=0.5$ KPa has been chosen small enough so that it does not affect the computational result.

At the free surface ($z=S$), the boundary condition consists of vanishing shear traction, so that

$$\tau_S=n_x\tau_{xx}+n_y\tau_{xy}+n_z\tau_{xz}=0. \quad (4)$$

At the basal surface, the form of the boundary conditions depends on the local conditions that are considered. In the present work basal sliding is introduced by relating the sliding speed u_b to the shear stress acting on the bed τ_b (Weertman, 1964; Lliboutry, 1968, 1979),

$$u_b=c\tau_b^{n'}, \quad (5)$$

with

$$\tau_b=n_y\tau_{xy}+n_z\tau_{xz}, \quad (6)$$

where $c=50$ m a^{-1} (MPa)$^{-1}$, is the sliding coefficient which is constant across the glacier bed, and has been chosen to obtain values for sliding velocity that allow for glacial erosion. We assume $n'=1$. At the margins, the flow speed on the surface is constrained to be zero.

2.2. Erosion model

Glacial erosion consists of several processes, including abrasion, plucking, subglacial fluvial erosion, and chemical dissolution by subglacial water. The highly complex nature of these mechanisms has precluded the development of physically complete models for these processes, and only a few empirical and theoretical analyses have provided indication of the primary controls on rates of glacial erosion. In the simulation described here, erosion rate normal to the bedrock surface is calculated as

$$E=Cu_b^2, \quad (7)$$

where C is an erosion constant equal to 10^{-4} a m^{-1} (Harbor, 1992; MacGregor *et al.*, 2000). Equation (7) is used here because it represents the general form of the abrasion law proposed by Hallet (1979).

2.3. Valley evolution model

As the initial glacier and valley geometries, we prescribed a V-shaped cross section with maximum ice thickness of 480 m, surface width of 1200 m, and downglacier slope of 4° (Fig. 2a). To solve Equation (1) for the flow speed within this cross section, a two-dimensional 35×35 finite-difference grid was employed. A set of finite-difference equations was solved with the LU factorization method assuming that the fluidity was constant and that the sliding speed was zero. Then, the computed velocity field was used to solve Equation (3) for the fluidity with a Newton-Raphson scheme so that the new values of the fluidity were used in the next iteration step. The velocity field was also used to compute the stress field with Equation (2) to introduce sliding speed in the next step using Equation (5). The computation was iterated until the velocity field converges within 2×10^{-4} m a^{-1}.

The flow model was coupled with the erosion model to investigate the temporal evolution of glacial valleys. For the first time step, the above procedures were used to calculate a flow pattern for the initial V-shaped valley. Calculated basal sliding speeds were substituted in Equation (7) to compute the pattern of erosion rate across the profile. Then the model calculates new coordinates for the glacier and the valley cross-section, which were used for the next time step. The ice surface elevation was calculated in a way that the total ice mass was kept constant during the simulation. However, for the simple model, the total ice mass was allowed to increase during the simulation, due to the particular high erosion rates at the valley center (Fig. 2c, d). These procedures were repeated for a given number of time steps.

3. Results and discussion

The present study aimed to show the importance of the lateral drag (τ_{xy}) included in Equation (1) and Equation (6), respectively. Therefore to highlight its influence on the formation of U-shaped valleys, we first ran a simple model, which neglects the lateral drag. Omitting the first term, integration of Equation (1) gives the basal shear stress as

$$\tau_b=-\rho gH\frac{\partial S}{\partial x}, \quad (8)$$

where H is the ice thickness. As shown in Figure 2b, the pattern of the sliding velocity for the initial V-

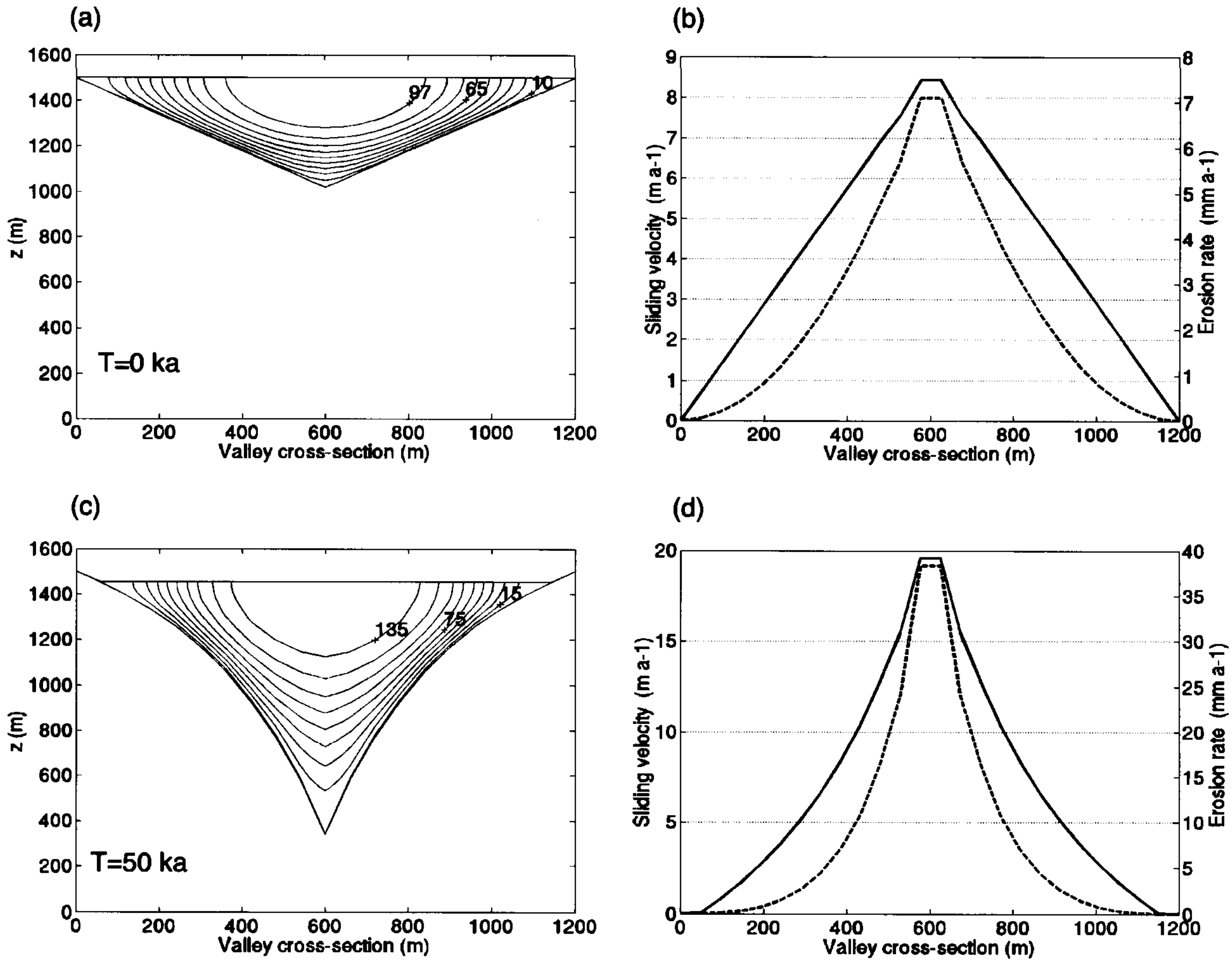

Fig. 2. (a) (c) Initial V-shaped valley and eroded valley obtained with the simplified model, velocity contours in m a $^{-1}$. (b) (d) Corresponding cross-glacier variation of sliding velocities (full line) and erosion rates (dotted line).

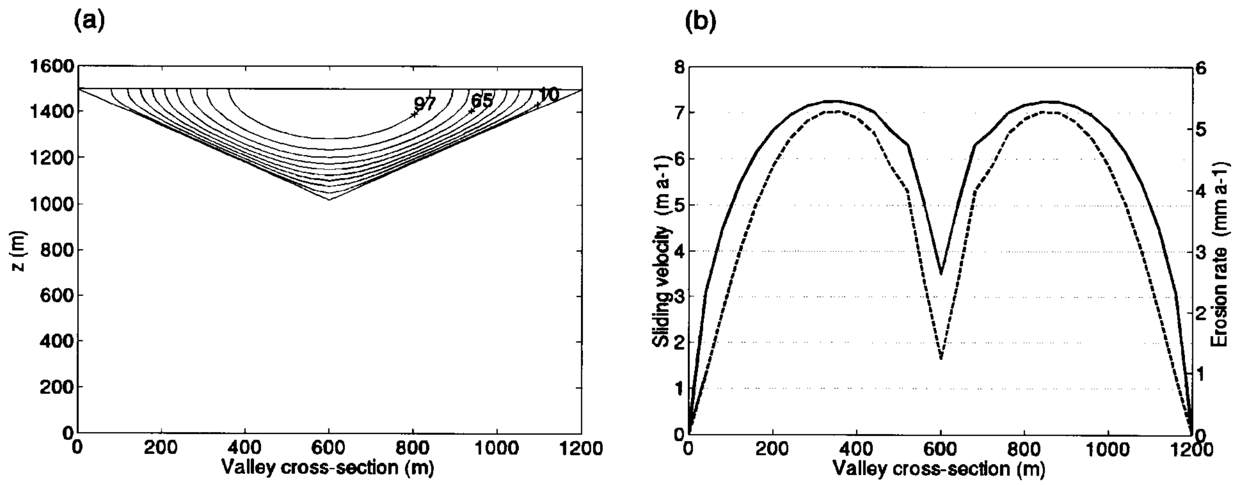

Fig. 3. (a) Initial V-shaped valley cross-section, velocity contours in m a $^{-1}$. (b) Corresponding cross-glacier variation of sliding velocities (full line) and erosion rates (dotted line).

shaped profile is represented by a rapid increase in the sliding speed toward the center of the valley and a decrease toward the margins. Because of the nature of the erosion law, erosion values follow the same pattern (Figs. 2b and 2d). This pattern of erosion was applied to the initial V-shaped valley, and Figure 2c shows the glacial valley eroded during the simulation for 50 ka. As observed, the valley shape remains similar to a V-shaped profile, characterized by a deep channel at the valley center, without the development of a U-shaped profile.

The results of the simplified model were com-

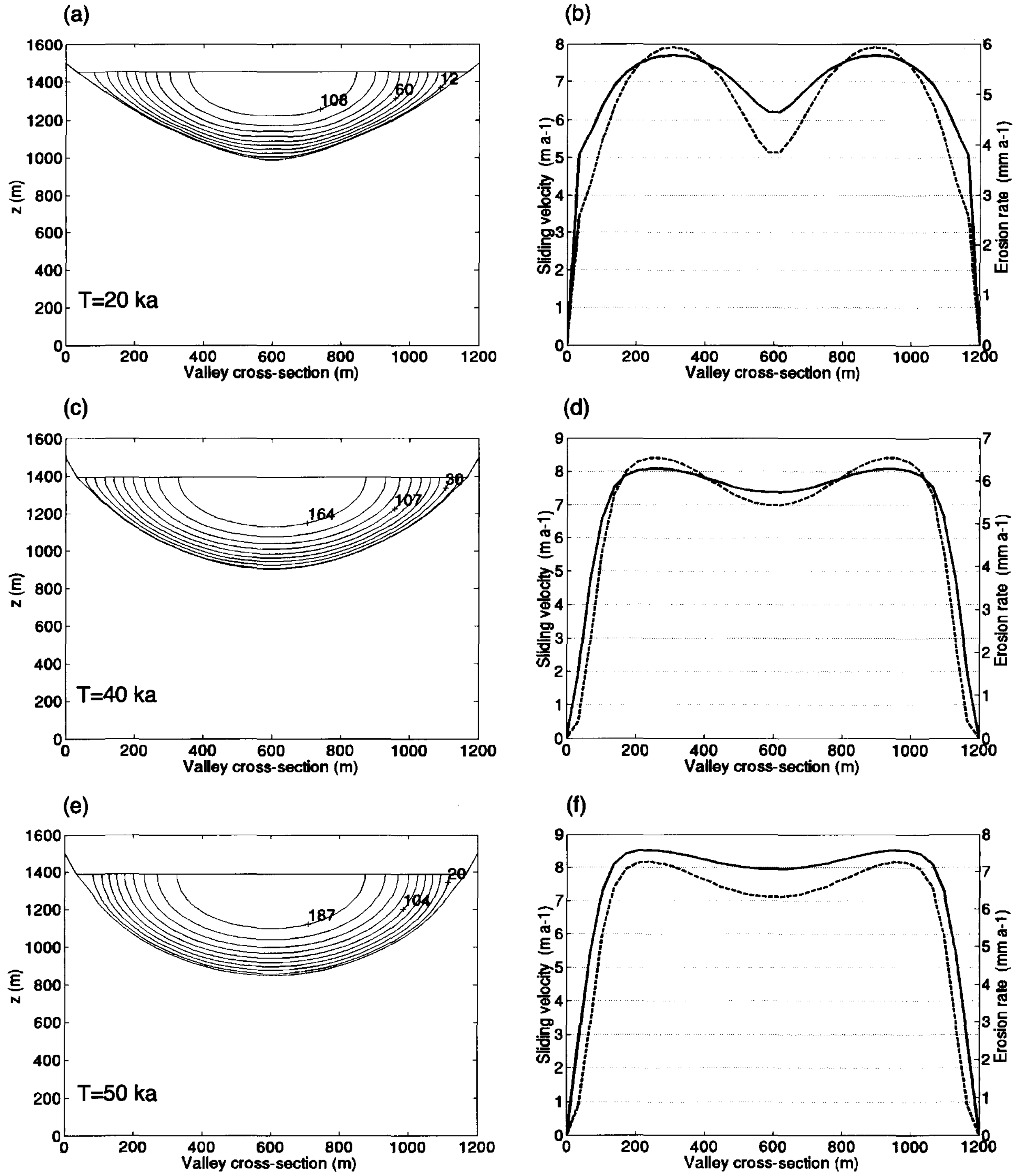

Fig. 4. (a) (c) (e) Model results for a simulation of glacial-valley development. Glacial-valley cross-section at three different time-steps, velocity contours in $m\ a^{-1}$. (b) (d) (f) Corresponding cross-glacier variation of sliding velocities (full line) and erosion rates (dotted line).

pared with those of the model which includes lateral drag. Figures 3a and b show the initial V-shaped valley, and the computed sliding velocity and erosion rate across the valley. Characteristic features of the sliding velocity and the erosion rate are increases toward the interior of the glacier, but with local minima at the center of the cross profile. The increase of drag associated with the constricted form at the center of the V-shaped valley reduces velocities there, resulting in a central minimum for the erosion as represented in Figure 3b. This is precisely the erosion pattern required to convert a V-shaped valley to a U-shaped channel, which do not appear in the simplified model (Fig. 2b). This comparison shows clearly the importance of the drag from the side walls (τ_{xy}) for the modeling of glacial erosion in a valley-cross section. Inclusion of both shear stress components τ_{xy} and τ_{xz} was necessary to obtain the sliding velocity pattern required for U-shaped valley formation. Figures 4a, c and e show the evolution of a cross-section valley for three different time-steps. The model predicts the evolution of the V-shaped profile into a recognizable glacial form with sliding velocities ranging from 3 m a^{-1} to 8 m a^{-1} (Figs. 4b, d, f) and for the last time-step, the model could simulate the formation of a deep and well-defined U-shaped valley (Fig. 4e). Although the time scale is sensitively dependent on the value of the erosion constant in the erosion law, the model suggests that a glacial valley can be developed after 50 ka or during a single glaciation under the condition of realistic sliding speed. This observation agrees with previous results from Harbor (1992). As the valley is progressively transformed into a U-shaped form, the central minima in velocity and erosion tend to be removed and disappear at the last time-step (Fig. 4f).

4. Conclusion

The present study described the development of a two-dimensional model, which has been successfully used for the simulation of glacial-valley evolution. A realistic U-shaped valley was obtained and the importance of lateral drag in the development of glacial-valleys was shown by the numerical experiments. To improve the modeling of glacial valley formation, it is important to know accurate values of ice flow and erosion parameters. Comparison of modeling results with field data of valley formation will provide an opportunity to constrain those parameters.

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