Bulletin of Glaciological Research 19 (2002) 93−99

On the intensity of ice melting in supraglacial and englacial channels

Evgeny ISENKO[1,2] and Bulat MAVLYUDOV[2]

1 Institute of Low Temperature Science, Hokkaido University, Sapporo 060-0035 Japan

2 Institute of Geography of Russian Academy of Sciences, Moscow, Russia

(Received October 22, 2001 ; Revised manuscript received November 19, 2001)

Abstract

The growth of a glacial channel that is filled with water both completely and partially is considered. The intensity of deepening of channels partially filled with water and the expansion of the completely filled channels are calculated by similar formulas. The influence of channel meandering and their cross-section nonroundness are also taken into account. The results of calculations turned out to be in agreement with known field data. Because we do not consider cold glaciers and plastic deformation of ice, this work can be thought as an estimation of the maximum values of channel growth rates. Taking into account only the potential energy of water, we obtained rather tolerable results. Therefore, this simple approach can be considered as a rude estimation of ice melting rate on the channel walls. In addition, a simplified model of a glacial ice-dammed lake was proposed. According to this model, most of the lake water usually drains through the dam during only several days, when the outburst occurs.

1. Introduction

During intensive ice melting, many small and large streams flow on the surface of glaciers. Such systems of supraglacial streams are usually arborescent (Sugden and John, 1976). Englacial drainage systems are also reckoned to be arborescent (Fountain and Walder, 1998). Obviously, in the glacial drainage system there should be the parts without tributaries or with negligible influence of them, which can be considered as a single channel. This work has purpose to describe the growth of such single channels both inside and on the surface of temperate glaciers.

In cold glaciers, a part of energy would be spent for ice warming and consequently the rate of channel growth would be less than that of temperate glaciers. The same situation is in englacial channels due to the ice deformation. Because here we do not take into consideration ice creep and ice warming, this work can be also considered as an attempt to estimate the maximum values of channel growth rates.

In spite of taking into account only the potential energy of water (including the potential energy of water pressure) and neglecting plastic deformation of ice, solar radiation, heat exchange with air etc., we insist on the applicability of this approach for many natural cases as a rude estimation of channel growth (deepening) rates in ice. There are many natural conditions, under which our approach would be rather tolerable. For example, the growth of the englacial conduits deep in the ice depends on the ice melting rate and the rate of the conduit closure under the weight of overburden ice (Rothlisberger, 1972). However, in the case of large water discharge the energy dissipation inside the water flow is so high that the resulting growth rate of the channel becomes significantly higher than that of plastic deformation of ice. We will describe this and another situations in detail in the next chapter.

Unfortunately, there are very few works describing the behavior of glacial channels (especially the changing of their geometrical parameters) and at the same time presenting some field measurements of their parameters and environment conditions. Therefore, we encountered some difficulties in comparison the obtained results with the field data to testify the model because of scarcity of the data, and are looking forward to have more measurements of glacial channels in various conditions.

2. Physical background

As flowing down a glacier, water gradually creates some drainage system. At the initial stage of its formation, for example at the beginning of an ablation period, the quantity of water is enough for filling up the most of the inner conduits. It means that the conduits of the englacial drainage system are usually completely filled with water (under pressure) at this time. As the conduits become bigger, the flow capacity of the drainage system increases. At some moment, the water inflow into the drainage system would not be sufficient to fill the conduits completely. Thus, at first, the biggest conduits and then smaller ones would become open channels and water would flow only through the lowest parts of their cross-sections. High water discharges usually accompany with high rates of ice melting. Consequently, the drainage conduits can become open again. That is why the englacial conduits are usually open when the channel slope or the water discharge is rather large (Hooke, 1984). In any case, we can subdivide the growing process of englacial conduits into two parts: completely and partially filled by water, or pressure and open channel regimes.

Let's consider now what physical factors cause glacial channels growth and what is the energy supply for ice melting on the channel walls.

Water loses its potential energy as flowing along a channel. Strictly speaking, we consider the potential energy as the sum of the gravitational potential energy and that of the water pressure. Obviously, in open channels, the water pressure term is absent. The lost potential energy is spent for an increase of the kinetic energy of the water flow and the energy dissipation inside the flow. The heat, which dissipates in the water flow, is spent in turn for water warming, ice melting and ice warming on the channel walls. It is clear, that some other natural factors, for example river deposits, can influence to the heat balance between ice and water, but we do not take them into account in this work.

At first, let's consider for simplicity a single straight channel with the same flow cross-sectional area along it. It means that the kinetic energy of water is constant along the conduit. Nevertheless, even in real channels with complicated forms, in which water flow velocity can change along the conduit, the influence of the kinetic energy inconstancy is negligible if the channel is sufficiently long. It can be clearer from the following example. If we suppose that the entire potential energy changes into the kinetic energy (that is obvious exaggeration), then only after 10 m of lowering, the water flow velocity would be 14 m s^{-1} that is rather large value even for huge streams. In the nature, the water velocity more than several meters per second is rare. Thus, if the height difference between the opposite points of a channel is much more than several meters, change in the kinetic energy of water is negligible comparing with change in its potential energy.

Let's suppose that some part of the energy is spent for water warming. Because the heat flux from water to ice is proportional to the temperature difference between ice and water, this heat flux would increase as water temperature became higher. Thus, the temperature of water cannot increase infinitely, but only up to some equilibrium value. Here we consider only such channels with equilibrium temperature conditions, when the water temperature is constant along the channel.

Strictly speaking, the temperature of the ice melting point depends on pressure. The pressure change involves a temperature change as well, to bring the water to the pressure melting point. Of course, it happens only in pressure (completely filled) channels. According to Rothlisberger (1972), about 1/3 of the energy is spent for such water temperature adjusting in a horizontal pressure conduit. For downsloping channels, this part of energy would be less and we do not consider this phenomenon here.

Finally, we do not take into account ice warming because only temperate glaciers are under consideration.

On the basis of the abovementioned argumentation we can conclude that the change in the potential energy ΔE_p is immediately spent for ice melting ΔE_i:

$$\Delta E_p = \Delta E_i, \tag{1}$$

Such approach is not original and was considered by other researchers too (Rothlisberger, 1972; Cutler, 1998), but we should emphasize that it is applicable only for a channel as a whole or significant part of it rather than for small parts of a channel, because, as we mentioned above, we cannot neglect the changing of the kinetic energy in short conduits.

Of course, such approach, when only the potential energy of water is taken into account, cannot be universally applicable. We need to say that it is not correct to apply this model for cases of high influence of warm weather conditions (warm air, solar radiation or precipitation), relatively high rates of ice deformation (small streams deep in the ice), presence of deposits in channels etc. Therefore, we do not consider these situations in this work.

Such simplified conditions are not rare in nature. If a surface channel is even slightly deepened into the ice, the influence of solar radiation and air temperature may become negligible. If a channel is deep in the ice but the water flow discharge is sufficiently high (especially during a glacial lake outburst), the ice creep may be negligible and so on.

3. Ice melting rate in open channels

Open channels can exist both on the surface and inside a glacier. In the latter case, we are in the situation that the solar radiation is absent and the influence of air temperature is negligible. Let's consider a straight channel with the same cross-section along it (Fig. 1) and let's take some water volume V with thickness Δx and the ice-contact surface area S_{wall}. We especially consider a channel with an arbitrary cross-section and water level to show universal

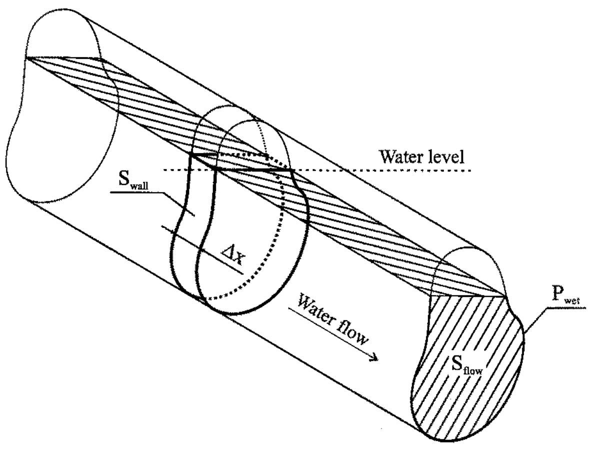

Fig. 1. The sketch of a straight inclined channel for understanding of the terms of the basic equations (2-4).

applicability of the argumentations. The water flow could be under pressure too, but such case we consider in the next chapter. The energy needed for melting of the ice with mass Δm equals

$$\Delta E_i = q\Delta m = q\rho_i S_{wall}\Delta r, \quad (2)$$

where q is the ice melting latent heat, ρ_i is the ice density and Δr is the thickness of melted ice on the channel walls. If this volume of water had moved down through the distance $v\Delta t$, , where v is the water flow velocity, its potential energy would be changed by the value

$$\Delta E_p = \rho_v V \cdot g \cdot v \cdot \Delta t \cdot \sin\alpha, \quad (3)$$

where ρ_v is the water density, g is the acceleration due to gravity, α is the channel slope. From simple geometrical considerations, the next relations are obvious:

$$S_{wall} = P_{wet} \cdot \Delta x, \quad (4a)$$

$$V = S_{flow} \cdot \Delta x, \quad (4b)$$

where P_{wet} is the wetted perimeter (the bold line on the face cross-section in Fig. 1), S_{flow} is the area of water flow cross-section (the hatching on the vertical surface). Combining the equations (1-4) and considering that the relation between the water flow cross-section area S_{flow} and the wetted perimeter P_{wet} is the hydraulic radius R (by definition), we find out the next equation for the ice melting rate:

$$\frac{dr}{dt} = \frac{\rho_w}{\rho_i}\frac{gR}{q} \cdot v \cdot \sin\alpha. \quad (5)$$

For round or half round conduits, the hydraulic radius equals to 1/4 of its diameter. The water flow velocity for steady-state conditions can be calculated by the well-known Shezi equation (Brebbia and Ferrante, 1983; Kiselev, 1972):

$$v = C\sqrt{R \cdot i}, \quad (6)$$

where i is hydraulic slope, which equals to $\sin\alpha$ for open channels; coefficient C is defined by Manning equation:

$$C = \frac{1}{n}R^{1/6}, \quad (7)$$

The friction coefficient n for clear ice is regarded to be approximately 0.02 (Rothlisberger, 1972). But for most of the real channels the value $C=40$ can be considered as constant, because for the hydraulic radius in the limits 0.025–2.5 m (that corresponds to a round tube with the diameter 0.1–10 m) the constant C varies in much less range: 27–58. Thus, a new equation for ice melting rate would be as follows:

$$\frac{dr}{dt} = \frac{\rho_w}{\rho_i}\frac{g}{q}C(R\sin\alpha)^{3/2}. \quad (8)$$

If we compare the ice-melting rate calculated by the equation (8) with some field data, we will find out that the equation (8) gives too high values. It is not surprising, because the real channels are not straight usually. Meandering of a channel means that for the same height difference the channel is longer by, say, k times, and we need to write $(1/k)\sin\alpha$ instead of $\sin\alpha$. For some glacial channels (in particular for surface ones), the meandering coefficient k can be defined directly. We can only suppose that channels inside glaciers have similar meandering coefficient.

According to Marston (1983), the meandering coefficient (sinuosity), measured on some glaciers of Alaska, was turned out to be equal to about 1.05–1.25. In Garver *et al.* (1994) the supraglacial channel with sinuosity 1.43 was called as gently sinuous. Our own observations of surface streams on Fisht Glacier, West Caucasus, gave the value about 1.5 and even 2 in some parts of the streams. So we will consider below that the channel has sinuosity 1.5, but have in mind that as applying the equations to some real glacial conduits, one should use its own value of meandering coefficient.

Another reason of disagreement between the measured and calculated values is the complicated form of channel cross-sections. It is obvious that the heat flux between ice and water depends on the ratio between the water volume and the area of the channel walls through which the heat exchange occurs. From a two-dimensional point of view, the heat flux depends on the ratio between the area of water flow cross-section and the wetted perimeter, i.e. the hydraulic radius. A round conduit is known to have the maximum value of the hydraulic radius. Therefore, if the cross-section of a channel was not round, the hydraulic radius would be smaller and, according to the equation (8), the ice-melting rate would decrease.

In summer of 1998, we observed small surface streams on Fisht Glacier, West Caucasus. The width of such channels was found to be about four times larger than its depth:

$$D/h \approx 4.$$

Probably, for others channels or some specific conditions this relation can be different. If we approximated such form of the conduit cross-section as an ellipse, the hydraulic radius would be as below:

$$R=\frac{S_{ellipse}}{P_{ellipse}} \approx \frac{D}{6}. \qquad (9)$$

The observations also show that even under sufficient diurnal oscillations of water discharge, the streams on the surface of the glacier tend to lower without changing its form rather than widen (Marston, 1983), i.e. both the width D and depth h remains constant. It means that for open channels the rate of ice melting is nothing else but the rate of the channel lowering.

Now we can write down the final equations for the ice-melting rate both for round and elliptical conduits with meandering:

$$\frac{dr}{dt}=\frac{\rho_w}{\rho_i}\frac{g}{q}C\left(\frac{D}{4k}\sin\alpha\right)^{3/2}, \qquad (10a)$$

$$\frac{dr}{dt}=\frac{\rho_w}{\rho_i}\frac{g}{q}C\left(\frac{D}{6k}\sin\alpha\right)^{3/2}. \qquad (10b)$$

The curves of the equation (10b) for different values of the channel slope α and the channel width D are shown in Fig. 2. We can see that the ice-melting rate in glacial streams is in the limits of 10-20 cm day^{-1} except very large or steep conduits.

Let's compare some field data with ones presented in Fig. 2. According to Marston (1983), the channel deepening rate for some glaciers in Alaska was in the range 4-8 cm day^{-1} that corresponds to rather realistic channel parameters: for example, the channel width 0.5 m, the slope 5-10°.

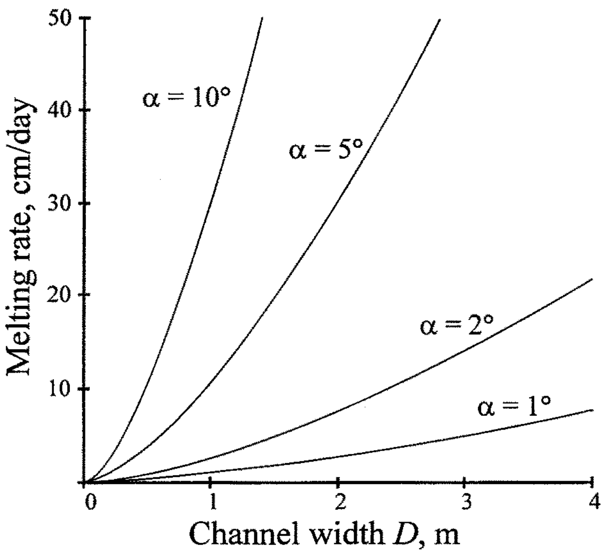

Fig. 2. Dependence of deepening rate for some open channel upon the channel width D for different value of α.

In 1998, at Fisht Glacier, West Caucasus, we took some measurements of deepening rate of surface streams (the ice melting rate at the bottom of them). The slope of the glacier was about 10-15°. The channel widths were from 10 cm for the smallest streams and up to 0.5 m for the biggest ones. This rate was found to be approximately equal to 4-7 cm day^{-1} at the daytime, but the water flow discharge and consequently the ice melting were much smaller at night. There were several measurement points in two channels of 40 and 20 cm width. These channels were deep sufficiently to exclude the influence of solar radiation. Four days averaging results in the ice melting rate 3.5, 4.3 (wide channel) and 2.2 (narrow channel) cm day^{-1} respectively. Because of the water level oscillations, the hydraulic radius varied from approximately zero to $D/6$ (see equation 10b). Therefore, the averaged value of the hydraulic radius was thought to be $R \approx D/12$. The curve of the equation (10b) for the above-mentioned conditions with the hydraulic radius correction and the measured values of the channels deepening are shown in Fig. 3. Although there are no many field data, we can see that our approach gives rather tolerable results.

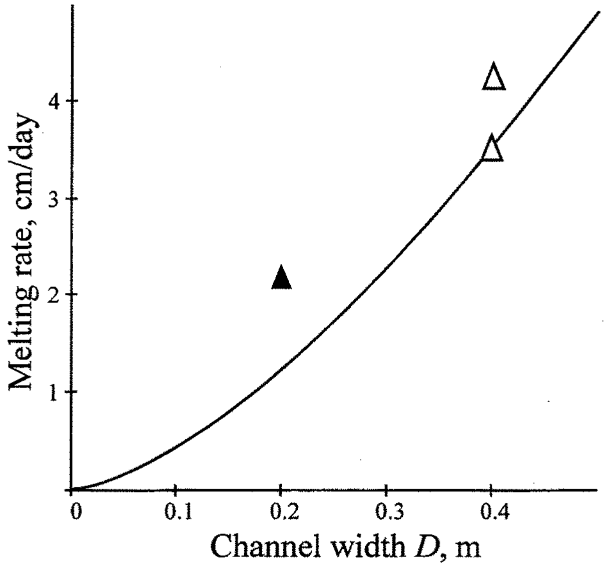

Fig. 3. The ice melting rates on the channel walls of the surface streams at Fisht Glacier in comparison with calculated values. The slope: 12°, the channel width: 40 and 20 cm, the ice melting rate: 3.5, 4.3 (white marks) and 2.2 (black mark) cm day^{-1} respectively (four days averaging).

4. Simplified model for glacial lake outburst

Lakes in glacial regions can commonly be formed near a glacier, on the surface of a glacier, or even inside of it, and may be dammed by moraines or ice. Whether a glacial lake is a stable system or not, depends on the lake water storage, the slope of the glacier, the type of the dam and so on (Raymond and Nolan, 2000). If by some reason a glacial lake system became unstable, the lake outburst could occur. The way, by which the water drains through the dam, is obviously different for ice and other material (moraine, earth dam, etc). Here we consider only ice -dammed lakes, where outbursts occur due to the ice melting. In addition to that, we consider only the situation, when the drainage conduit is completely inside the glacier (Fig. 4). Such situation, for example, takes place for the well-known huge Mertzbacher Lake on Inyltchek Glacier, Tien-Shan (Mavlyudov, 1996).

At first, we should notice that in case of ice -dammed lake draining, it should be one trunk channel instead of an arborescent system. Secondly, we propose that the channel is under pressure along all its length, i.e. probably like in the drainage channel of Mertzbacher Lake. Because pressure conduits tend to be round in its cross-section (Isenko, 2000), we can apply equation 10a to our model, but of course such an approach turns out to be only a rude approximation.

One should remember that the potential energy of water in pressure conduits is not only the gravitational potential energy, as we considered before, but the sum of it and the energy of the water pressure. That is why we have to write down the hydraulic slope i instead of the geometrical $\sin\alpha$ slope (see for example Shreve (1972), Kiselev (1972)):

$$i=\sin\alpha+\frac{1}{\rho_w g}\frac{\Delta p}{\Delta x}, \tag{11}$$

where $\Delta p/\Delta x$ is the pressure gradient inside the conduit.

So let's consider a water reservoir and a conduit started from the wall of the reservoir at some depth h_0 from the water surface. As passing from the lake to the conduit, the water increases its velocity from approximately zero to some value v. It means that the water pressure drops sharply near the conduit entrance. Therefore, water pressure along the dashed line in Fig. 4 at first increases proportionally to the depth of the lake, then drops sharply near the conduit entrance, and further, inside the conduit, water pressure decreases linearly to zero (strictly speaking, to air pressure). The pressure jump near the conduit entrance can be defined by Bernoulli equation:

$$\frac{v^2}{2}+\frac{p}{\rho_w}=const. \tag{12}$$

As was mentioned above, along the streamline near the conduit entrance, the water velocity increases approximately from zero to v. Therefore, according to (12), the water pressure decreases by the value $\Delta p=\rho_w v^2/2$ and the water pressure at the upper (entrance) point of the conduit becomes as below:

$$p_0=\rho_w g h_0-\frac{\rho_w v^2}{2}. \tag{13}$$

In terms of the model in Fig. 4, equation (11) would be as follows:

$$i=\frac{h}{l}+\frac{h_0-v^2/2g}{l}. \tag{14}$$

Combining this equation with Shezi formula (6) and extracting the hydraulic slope i from it, we find:

$$i=\frac{2g(h+h_0)}{2gl+C^2R}. \tag{15}$$

Finally, combining equation (15) with (10a), we obtain a new equation for ice-melting rate in round pressure conduits in terms of the glacial lake drainage model:

$$\frac{dr}{dt}=\frac{\rho_w}{\rho_i}\frac{g}{qC^2}\left[\frac{\frac{4}{3}g(h+h_0)}{\frac{8gl}{DC^2}+1}\right]^{3/2}. \tag{16}$$

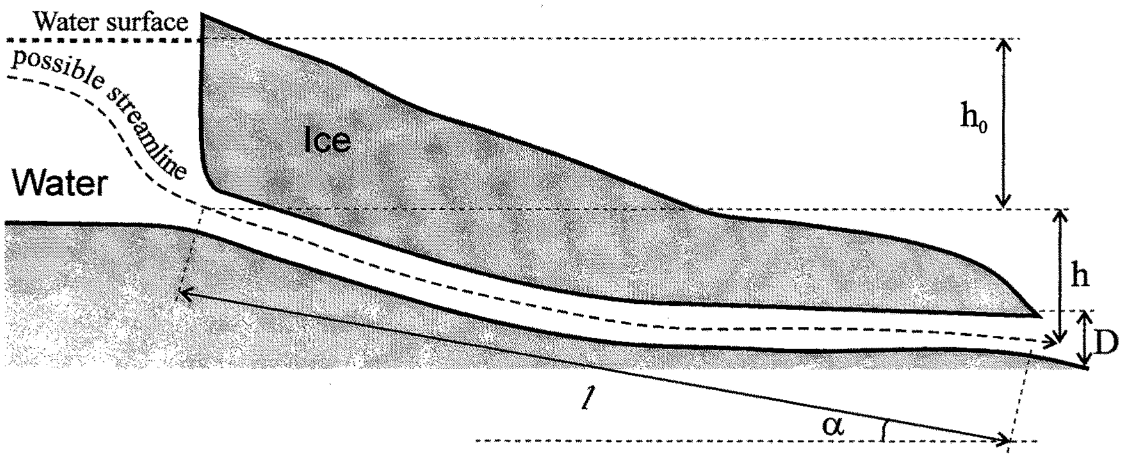

Fig. 4. Schematic diagram of a water reservoir with a drainage channel. h_0: the depth of the reservoir, h: the height difference between ends of the channel, l, D and α: the length, the diameter and the slope of the channel, respectively. A possible water stream-line is shown by the dashed line.

The curves of equation (16) are shown in Fig. 5 for different parameters. We can see (curves 2 and 4) that the channel growth behavior is almost the same as the deepening of open channels (Fig. 2). It is because for most of glacial channels the value

$$\frac{8gl}{DC^2} \gg 1, \tag{17}$$

and the growth rate is proportional to the 3/2 power of the diameter, like for open channels. Only when the ratio between the channel length and its diameter became sufficiently small ($l < \sim 50D$), then another type of behavior is appeared. The curve 3 in Fig. 5 is especially drawn for such conditions to show this effect. The line 1 is for some transient conditions ($l \approx 50D$). The curve 4 corresponds to the growth rate for the conduit, which drains ice-dammed Mertzbacher Lake. We should say that from mathematical point of view, if we enlarge the range of the chart area, then each curve in Fig. 5 would show such behavior as the curve 3. For example, the curve 1 would be like the curve 3 when the channel diameter reached 10-15 meters, but such values are seemed to be nonrealistic.

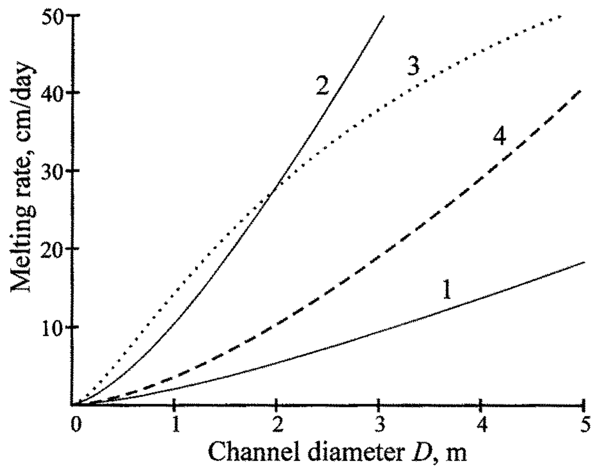

Fig. 5. Dependence of the channel growth rate upon its diameter for the ice-dammed lake model with some values of the channel length l and the altitude difference $h + h_0$ between the lake surface and the water outlet. **1**: $h + h_0 = 10$ m, $l = 500$ m; **2**: $h + h_0 = 30$ m, $l = 500$ m; **3**: $h + h_0 = 5$ m, $l = 50$ m; **4**: $h + h_0 = 400$ m, $l = 14000$ m. See explanations in the text.

To find out the dynamics of the channel growth, we have to solve the differential equation (16) for the value D. Here we should remember that $dD/dt = 2dr/dt$ and notice that unlike the channel width D from the previous chapter, where D is constant, the channel diameter D for the pressure conduit is variable.

Because equation (16) has no analytical solution, we will solve the simplified equation using the condition (17). It gives us the following function:

$$D(t) = \frac{1}{\left[\frac{1}{\sqrt{D_0}} - at\right]^2}, \tag{18}$$

where D_0 is the initial conduit diameter (at the moment $t=0$) and a is a constant as:

$$a = \frac{\rho_w}{\rho_i}\frac{g}{q}C\left[\frac{h+h_0}{6l}\right]^{3/2}. \tag{19}$$

Figure 6 shows the channel growth rate for the parameters of Mertzbacher Lake according to equation (18). We can see that the channel diameter grows very slowly at the initial stage, but then its size becomes grow very fast and it reaches sufficient values during only several days. Of course, it will continue only until the lake is depleted. This can explain the flood regime of a glacial-dammed lake outburst when the long period of low water discharge is followed by relatively short period of lake drain. Mertzbacher Lake, for example, drains completely once a year during approximately 8 days after the beginning of outburst.

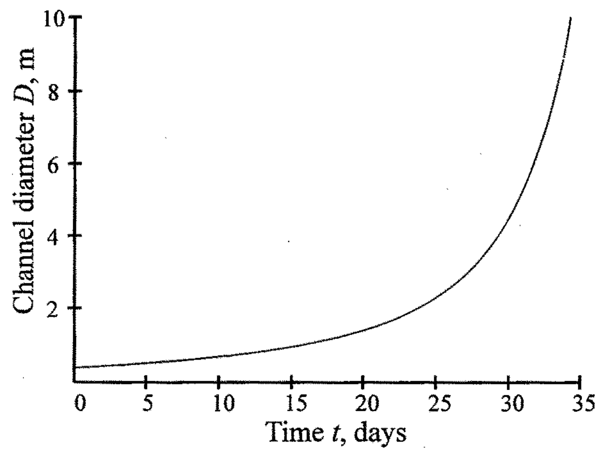

Fig. 6. The increase of the channel diameter with time, calculated by of the equation (18). The parameters are the same as of the channel, which drains out ice-dammed lake Mertzbacher on Inyltchek Glacier.

We should say that this model correctly describes only the final stage of a glacial lake drain, after the outburst begins. At the time before it, the water flow discharge is not high and consequently the situations, when the ice creep is not negligible, the conduit cross-section is not round or the water flow is not pressure along the conduit, probably take the place. Therefore, this model cannot predict, for example, the time between outbursts or strict value of outflow hydrograph during usual period of drain (unlike outburst regime). On the contrary, the behavior (not strict value) of the flow hydrograph is seemed to be well described by the aforementioned technique. If by some reason the channel growth became higher its closure rate due to the ice creep (for example, because of increasing of the water flow discharge at the beginning of an ablation period), the conduit starts to grow, may be, very slowly at first. As the conduit is enlarging, the water flow discharge becomes higher and more and more parts of the conduit become completely filled with water (pressure), creating conditions appropriate for our model.

5. Conclusions

1. Consideration of only the change in the potential energy of a water flow for defining ice-melting rate on the channel walls gave rather realistic results. It means that for the most common glacial channels the potential energy is the most significant part in the energy balance inside a water flow and that such approach can be applicable to many of natural glacial streams.
2. Usually, the ice-melting rate in glacial streams is in the limits of 10-20 cm day^{-1} except very large or steep conduits.
3. Both the intensity of the deepening of the channels partially filled with water and the expansion of the completely filled channels are proved to be proportional to the 3/2 power of the conduit size, i.e. calculated by similar formulas.
4. We need to take into account conduits meandering and their cross-section nonroundness, otherwise it causes to overestimating of the ice-melting rate in several times.
5. On the basis of the abovementioned backgrounds, a model of a glacial ice-dammed lake was presented. The main result of this modeling is peak-like hydrograph of the outflow from the lake, when the long period of low water discharge is followed by relatively short period (several days or weeks) of lake drain.

Acknowledgments

We thank Dr. Renji Naruse (Institute of Low Temperature Science) for helpful comments and advices to this study. We are also gratitude to an anonymous reviewer for precious remarks and suggestions on the manuscript.

References

Brebbia, C. A. and Ferrante, A. J. (1983): Computational Hydraulics. Butterworth, London, 290 pp.

Cutler, P. M. (1998): Modelling the evolution of subglacial tunnels due to varying water input. J. Glaciol., **44** (148), 485-497.

Fountain, A. G. and Walder, J. S. (1998): Water flow through temperate glaciers. Reviews of Geophysics, **36** (3), 299-328.

Garver, S., Sear, D. and Valentine, E. (1994): An observation of roll waves in a supraglacial meltwater channel, Harlech Gletscher, East Greenland. J. Glaciol., **40** (134), 75-78.

Hooke, R. L. (1984): On the role of mechanical energy in maintaining subglacial water conduits at atmospheric pressure. J. Glaciol., **30** (105), 180-187.

Isenko, E. V. (2000): Modeling of channels in cold glaciers. Data of glaciological studies, **89**, 194-199 (in Russian).

Kiselev, P. G. (1972): Hydraulic reference book. Moscow, Energy, 312 pp (in Russian).

Marston, R. A. (1983): Supraglacial stream dynamics on the Juneau icefield. Annals of the Association of American Geographers, **73** (4), 597-608.

Mavlyudov, B. R. (1997): Drainage of the ice-dammed Mertzbacher lake, Tien-Shan. Data of glaciological studies, **81**, 61-65.

Raymond, C. F. and Nolan, M. (2000): Drainage of a glacial lake through an ice spillway. Debris-Covered Glaciers (Proceedings of a workshop held at Seattle, USA), IAHS Publ. no. 264, 199-207.

Rothlisberger, H. (1972): Water pressure in intra- and subglacial channels. J. Glaciol., **11** (62), 177-203.

Shreve, R. L. (1972): Movement of water in glaciers. J. Glaciol., **11** (62), 205-214.

Sugden, D. E. and John, B. S. (1976): Glaciers and Landscape. London.