

Heat transfer in the seasonal active layer of Gorshkov Ice Cap on the summit of Ushkovsky Volcano, Kamchatka Peninsula

Andrey N. SALAMATIN¹, Takayuki SHIRAIWA², Yaroslav D. MURAVYEV³ and Marat F. ZIGANSHIN¹

¹ Department of Applied Mathematics, Kazan State University, Kazan 420008 Russia

² Institute of Low Temperature Science, Hokkaido University, Sapporo 060-0819 Japan

³ Institute of Volcanology, Petropavlovsk-Kamchatsky, Kamchatka 683006 Russia

(Received October 17, 2001 ; Revised manuscript received November 21, 2001)

Abstract

Seasonal heat and mass transfer in the cold snow-accumulation area of Gorshkov Ice Cap (3903 m a.s.l.) on Ushkovsky Volcano is studied on the basis of model simulations to interpret continuous two-year temperature records from seven thermistors installed in the 27-meter deep borehole. Precipitation seasonality, relatively high annual accumulation rates, and snow-firn densification are found to be the principal peculiarities of the process. Only first harmonics of the seasonal temperature variations were significant and reliably detectable. The amplitude of the surface-temperature fluctuations is inferred to be 16.6°C. The mean annual surface temperature of 1996 is determined as -17.5°C with a short-term trend about $-2.3^{\circ}\text{C yr}^{-1}$ during the 1996–1998 period. The relative thermal conductivity of snow and firn λ as a parametric function of porosity c is simultaneously verified with the parameter $a \approx 0.5$. The impact of the interaction between the seasonal variations of the surface temperature and the mass balance rate on the thermal state of the glacier below the active layer is theoretically estimated as very small.

1. Introduction

Glaciers filling volcanic craters form unique natural complexes with specific thermal and hydrodynamic interactions. Comparatively large depth of the craters and small longitudinal velocities of the ice make such glaciers especially valuable as paleoclimatic and volcanic archives. Heat transfer processes in the seasonal active layer and the energetic capacity of the volcano manifest themselves through the thermal state and vertical movement (bottom melting) of the glacier, and determine the age-depth distribution of ice and ash deposits (Muravyev and Salamatina, 1989; Salamatina *et al.*, 2000; Shiraiwa *et al.*, in press).

Seasonal heat and mass transfer processes in the cold snow-accumulation area on Ushkovsky Volcano were studied from 1996 to 1998 in the framework of the Joint Russian-Japanese Cryospheric Research Project at Kamchatka (Kobayashi *et al.*, 1997; Shiraiwa *et al.*, 1997; 1999a; 1999b). Gorshkov Ice Cap (3903 m a.s.l.) filling the larger summit volcanic crater was the site of the field observations (Fig. 1) which included (1) continuous temperature records from the seven sensors hanging on one string and set at the initial depth levels around 1, 2, 4, 5, 8, 10 and 27 m in the 27

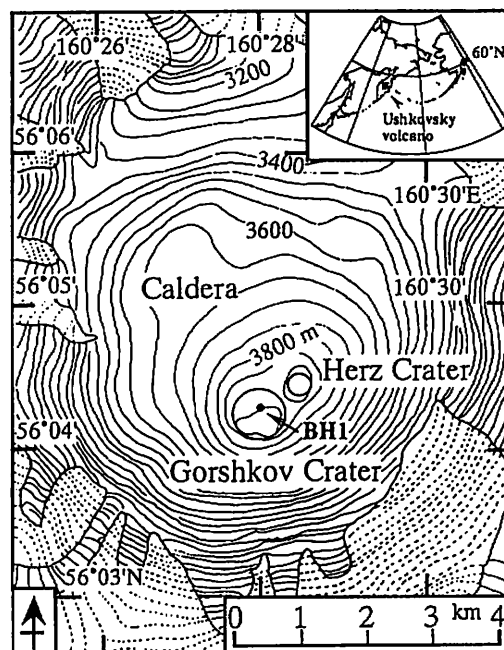


Fig. 1. Topographic map of the summit ice cap on Ushkovsky Volcano, Kamchatka, with the location of the 27-meter deep borehole BH-1 used for the seasonal active layer thermometry in 1996–1998. Solid and dotted contours are for the glacier and bedrock, respectively.

-meter deep borehole at BH-1 from August 1, 1996 till June 30, 1998 (Fig. 2); (2) annual measurements of snow-accumulation stakes; (3) ice-mass balance and density measurements in pits; and (4) ice-core analyses.

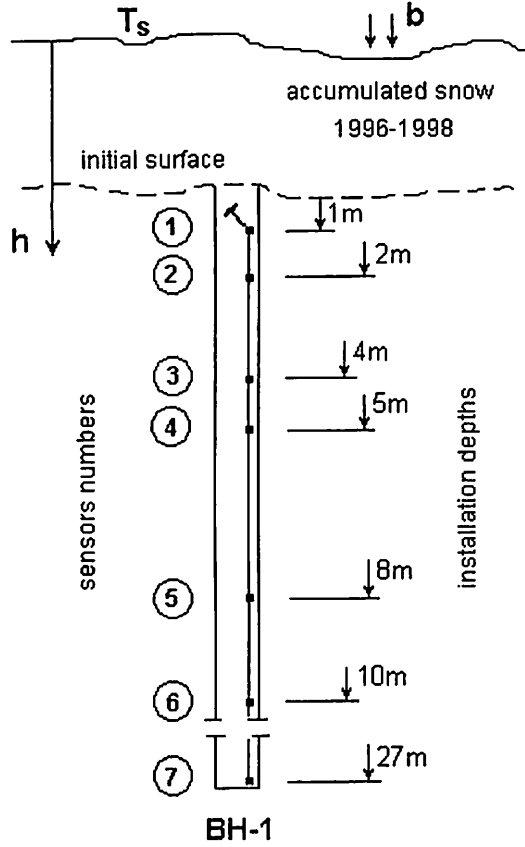


Fig. 2. A sketch of the borehole BH-1 with the installed thermistors moving in the snow-firn near-surface stratum.

The principal goal of this study is to interpret the observed temperature data obtained from the moving thermistors by means of model simulations on the basis of seasonality in precipitation, annual snow accumulation rates ($\sim 0.6 \text{ m yr}^{-1}$ in ice equivalent), and snow-firn densification processes.

2. Mathematical model

A mathematical model and a special algorithm are developed to simulate seasonal temperature variations T (in $^{\circ}\text{C}$) in a dry snow-firn near-surface strata of a glacier.

The boundary value problem is formulated for the general heat transfer equation in the half-space of the snow-firn deposits with the h -axis directed downward and depth h counted from the snow surface (Fig. 2).

$$c_{pi}\rho_i(1-c)\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial h}\right] = \frac{\partial}{\partial h}\left[\lambda_i\Lambda\frac{\partial T}{\partial h}\right] \quad (1)$$

Here t is the time, ρ_i , $c_{pi}(T)$, and $\lambda_i(T)$ are the density,

heat capacity, and thermal conductivity of pure ice, respectively; $c(h)$ is the snow-firn porosity; $\Lambda(c)$ is the normalized correcting factor which accounts for the enhanced thermal resistance of the porous ice structure; v is the vertical velocity of the snow and firn relative to the glacier surface. The main model parameters, their values and definitions used in simulations are compiled in Table 1. Thermophysical properties of ice are taken after Budd (1969) and Hobbs (1974). The relative thermal conductivity of snow and firn Λ is determined in accordance with Muravyev and Salamatina (1989) and Salamatina *et al.* (2000) after Vostretsov *et al.* (1984) and Sturm *et al.* (1997). Porosity profile along the borehole is obtained on the basis of the ice core density measurements of Shiraiwa *et al.* (1997).

Periodic temperature variations are imposed at the air-snow interface

$$T_{h=0} = \langle T_s \rangle + \sum_{k=1}^{N \leq 2} \left[A_k \cos\left[\frac{2k\pi}{t_0}t\right] + B_k \sin\left[\frac{2k\pi}{t_0}t\right] \right], \quad (2)$$

where $\langle T_s \rangle$ is the mean annual temperature, A_k and B_k , $k = 1, 2, \dots$, are the amplitudes of the harmonic oscillations, t_0 is the period (one year).

It is assumed that $T \rightarrow \langle T_s \rangle$, as $h \rightarrow \infty$, and the limiting periodic solution is searched for $t \rightarrow \infty$.

The principal point here is that, due to the seasonality in precipitation, the vertical velocity v of the snow and firn deposits also varies periodically with time, and in accordance with Salamatina (1991) and Salamatina *et al.* (2000) it can be presented as:

$$v = \frac{b - \langle b \rangle}{1 - c_s} + \frac{\langle b \rangle}{1 - c} [1 - (1 - \theta + \varepsilon)(1 - \zeta)],$$

$$\zeta = \int_h^{h_0} (1 - c) dh / \int_0^{h_0} (1 - c) dh. \quad (3)$$

The latter equation takes into account the effects of the snow-firn compressibility and the ice cap deformation. The following denotations are used: b and $\langle b \rangle$ are the current and mean annual mass balance (accumulation rates) of ice on the glacier surface, respectively; c_s is the mean porosity of the annual snow layer; θ and ε are the relative ice melt rate at the glacier bottom and the relative vertical compression rate of the ice, respectively, normalized by $\langle b \rangle$; h_0 is the ice-cap thickness; ζ is the normalized distance from the glacier (crater) bottom in ice equivalent. All environmental parameters are presented in Table 1 after Matsuoka *et al.* (1999), Shiraiwa *et al.* (1999b) and Salamatina *et al.* (2000). Mass balance fluctuations are expressed, similarly to Eq. (2), as

$$\frac{b}{\langle b \rangle} - 1 = \sum_{k=1}^{N \leq 2} \left[U_k \cos\left[\frac{2k\pi}{t_0}t\right] + V_k \sin\left[\frac{2k\pi}{t_0}t\right] \right], \quad (4)$$

where U_k and V_k are the normalized amplitudes of the

Table 1. Main model parameters

Meaning of parameters	Denotations	Basic values and formulas
<i>Thermophysical properties of glacier ice:</i>		
Ice density	ρ_i	918 kg·m ⁻³
Specific heat capacity	$c_{pi}(T)$	$c_{p0}[1 + \alpha_c(T + 30)]$, $c_{p0} = 1.89 \text{ kJ} \cdot (\text{kg} \cdot ^\circ\text{C})^{-1}$, $\alpha_c = 0.0037^\circ\text{C}^{-1}$
Thermal conductivity	$\lambda_i(T)$	$\lambda_0[1 - \alpha_l(T + 30)]$ $\lambda_0 = 2.55 \text{ W} \cdot (\text{m} \cdot ^\circ\text{C})^{-1}$, $\alpha_l = 0.0039^\circ\text{C}^{-1}$
Relative thermal conductivity of snow and firn*	$\Lambda(c)$	$a(1 - c)/(a + c)$, $a \approx 0.5$
Porosity	c	$c_s e^{-\gamma h}$ $c_s \approx 0.5$, $\gamma \approx 0.03 \text{ m}^{-1}$
<i>Environmental conditions:</i>		
Ice cap thickness	h_0	185 m
Relative melt rate	θ	0.1
Relative rate of vertical compression	ϵ	-0.6
Mean annual surface temperature	$\langle T_s \rangle$	-17.5°C
Mean annual mass balance	$\langle b \rangle$	0.61 m·yr ⁻¹
<i>Seasonal oscillations of climate characteristics:</i>		
Surface-temperature amplitudes*	A_1	15.7°C
	B_1	-5.4°C
Mass-balance relative amplitudes	U_1	0.34
	V_1	0.28
	U_2	0.49
	V_2	0.26

* Best-fit values deduced in this paper

mass balance seasonal fluctuations.

As shown below, the interaction between the surface-temperature and mass-balance variations given by Eqs. (2) and (4), even for $N = 1$, results in additional high-frequency harmonics in the temperature field T determined by Eq. (1).

3. Experimental data

Measurements of snow-accumulation stakes, observations in pits, ice core data, and ash markers of historical eruptions allowed us to reliably estimate the annual mass balance rate $\langle b \rangle \approx 0.6 \text{ m yr}^{-1}$ of ice averaged over 28 years (Shiraiwa *et al.*, 1997). The seasonal variations of the accumulation rate have also been deduced from the stratigraphic features of the 1994–1998 firn deposits and monthly precipitation data observed at Kluchi, a nearby meteorological station at the foot slope of the Ushkovsky Volcano. As a result, normalized histogram of the monthly mass-balance fluctuations is presented in Fig. 3a. It is periodically extrapolated over the two-year period of the continuous borehole-temperature measurements from August 1, 1996 till June 30, 1998. Hereinafter the starting date is the initial moment (zero time) of the time scale in our considerations. Thick and thin solid lines in Fig. 3a are the harmonic expansions of the mass balance oscillations in accordance with Eq. (4) for $N = 1$ and 2. They are referred further as cases

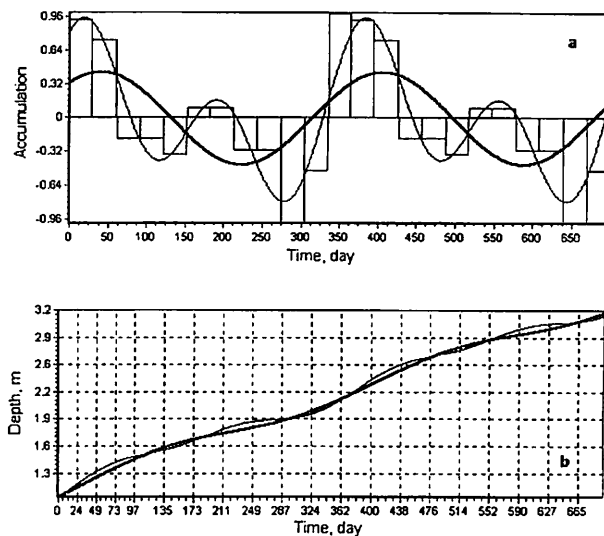


Fig. 3. (a) The normalized histogram of the mass balance monthly fluctuations periodically extrapolated over the two-year period of the continuous borehole-temperature measurements from August 1, 1996 till June 30, 1998. The starting date is the initial moment (zero time) of the time scale. Thick and thin solid lines are the harmonic expansions of the mass balance oscillations in accordance with Eq. (4) for $N = 1$ and 2 (see also Table 1). (b) The downward movement of the first upper sensor fixed in firn. The trajectories are predicted on the basis of Eqs. (3), (4) at $\langle b \rangle \approx 0.61 \text{ m yr}^{-1}$. The thick and thin lines correspond to $N = 1$ and 2 in Eq. (4), respectively.

I and II, respectively.

As schematically shown in Fig. 2, seven thermistors were installed in the borehole BH-1 at the initial depth levels of 1, 2, 4, 5, 8, 10, and 27 meters. Only the first one was fixed in the firm and reached the depth of 3.15 m by the end of the experiment. Its downward movement predicted on the basis of Eqs. (3) and (4) is depicted in Fig. 3b by the thick and thin lines for the cases I and II, respectively. The two variants of the trajectory are rather similar and come to the 3.15 meter depth at $\langle b \rangle \approx 0.61 \text{ m yr}^{-1}$. The latter value of the mass balance rate is very close to its above long-term estimate and is used in all simulations below. All other sensors hung freely on the same string and temporal variations of their depths were the obvious shifted replications of those of the first one.

Six original daily-temperature records after eliminating linear trends are plotted in Fig. 4 and can easily be distinguished due to the high-frequency noise induced by the measuring equipment. Numbers of the thermistors in the downward direction are given in circles. The last (seventh) sensor at the bottom of the borehole did not reveal any noticeable seasonal changes in temperature. Climatic instability of the surface-temperature variations and non-uniform increase in the depth of the sensors locations manifest themselves not only through the general non-periodic trends in the recorded signals but also distort their phase lags and damp their amplitudes with time. To determine the resulting short-term temporal changes in harmonic expansions of the temperature-time series, the measurement data are subjected to the multi-factor (harmonic) analysis and each temperature record $T_{ex}(t)$ is presented in the following form:

$$T_{ex}(t) = T_0 + c_0 t + \sum_{k=1}^{N \leq 2} \left[(A_k + \alpha_k t) \cos\left(\frac{2k\pi}{t_0} t\right) + (B_k + \beta_k t) \sin\left(\frac{2k\pi}{t_0} t\right) \right]. \quad (5)$$

The best-fit values of the parameters in Eq. (5) are given in Table 2. As it can easily be seen, only first harmonics are significant and reliably detectable. The amplitudes of double-frequency oscillations are 10 times less and comparable with the level of the noise (standard deviation σ).

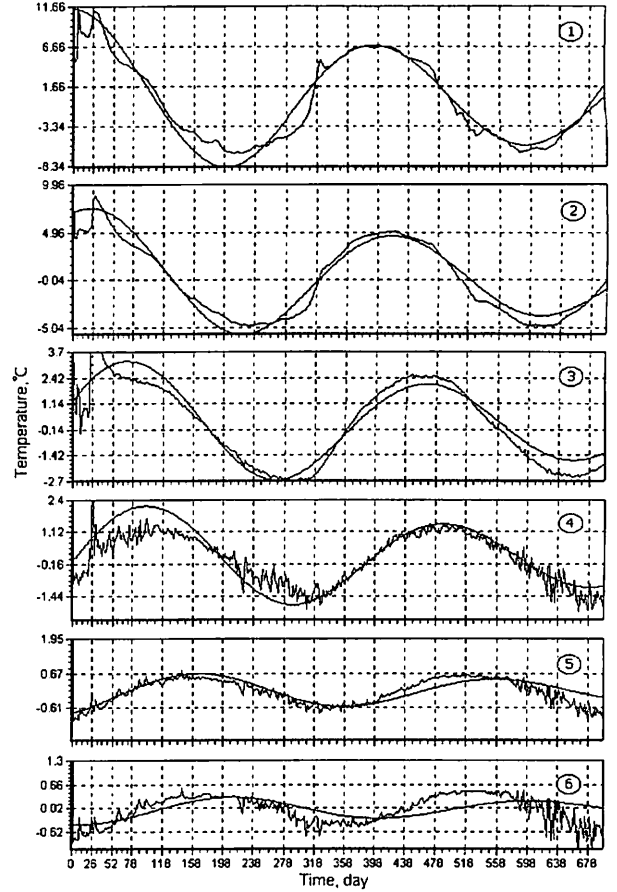


Fig. 4. Comparison of the best-fit smooth theoretical predictions with the six original daily-temperature records without linear trends easily distinguished due to the high-frequency noise induced by the measuring equipment. Numbers of the thermistors in the downward direction are given in circles.

4. Model simulations

To interpret the above quantitative data inferred from experimental measurements we have to separate the original climatic input from the impact caused by the non-uniform increase in the depths of the thermistor locations. With this aim, we directly simulated the sensor temperatures, using the model (1)–(4) at $N = 1$ and model parameters given in Table 1. The amplitudes of the surface-temperature fluctuations A_1 and B_1 in Eq. (2) and the relative thermal conductivity of

Table 2. Parameters of the harmonic analysis of the registered temperature signals

Parameters	Sensors						
	1	2	3	4	5	6	7
$T_0, ^\circ\text{C}$	-15.41	-15.14	-15.12	-15.25	-15.57	-15.63	-14.61
$c_0, ^\circ\text{C} \cdot \text{yr}^{-1}$	-2.19	-1.93	-1.24	-0.55	-0.25	-0.18	0.00
$A_1, ^\circ\text{C}$	7.36	4.33	0.83	-0.32	-0.45	-0.35	0.008
$\alpha_1, ^\circ\text{C} \cdot \text{yr}^{-1}$	-1.58	-1.02	-0.86	-0.31	-0.086	-0.085	-0.01
$B_1, ^\circ\text{C}$	3.06	3.62	2.87	1.15	0.13	-0.087	0.004
$\beta_1, ^\circ\text{C} \cdot \text{yr}^{-1}$	0.84	0.33	-0.27	-0.025	0.11	0.20	-0.006
$A_2, ^\circ\text{C}$	0.62	0.41	0.069	-0.055	-0.032	-0.014	0.001
$B_2, ^\circ\text{C}$	0.14	0.27	0.26	0.11	-0.0025	-0.0065	0.002
$\sigma, ^\circ\text{C}$	0.82	0.47	0.36	0.24	0.10	0.12	0.01

snow and firn $\Lambda(c)$ (the parameter a in Table 1) were adjusted to minimize the standard deviation between the model predictions and experimental measurements after subtracting the linear trends. The best-fit smooth theoretical predictions are compared in Fig. 4 with original temperature signals. The discrepancy for the four upper sensors has actually the same order of magnitude as the level of the experimental noise and does not exceed 2σ . This reliably constrains the surface temperature oscillations with accuracy of about $\pm 1^\circ\text{C}$. However, in spite of the obviously close amplitudes, a noticeable difference in phases between the simulated and measured temperature records for the two lower sensors results in much higher deviations. The latter disagreement may be attributed to some additional irregularities in the sensor motions caused by random anomalies in precipitation.

It is important to note that the simulated temperature field in the borehole is periodic in time and consists of the following components:

$$T(h, t) = T_0(h) + \sum_{k=1,2,\dots} \left[A_k(h) \cos\left[\frac{2k\pi}{t_0} t\right] + B_k(h) \sin\left[\frac{2k\pi}{t_0} t\right] \right]. \quad (6)$$

The non-periodic term T_0 counted from the mean annual surface temperature $\langle T_s \rangle$ and the total amplitudes of the harmonic oscillations $(A_k^2 + B_k^2)^{1/2}$, $k = 1, 2$, are plotted in Fig. 5a versus depth h .

As could be expected, the first harmonic dominates in the temperature variations, and its amplitudes at different depths practically coincide with those (see dots in Fig. 5a) deduced from the experimental measurements on average, at $t \approx 1$ yr in Eq. (4) (Table 2). The standard deviation is about 0.2°C . Thus, based on the comparison presented in Figs. 4 and 5a, we come to the principal conclusion that the detected temporal changes in amplitudes of the experimental records should mainly be attributed to the downward movement of the sensors and the climatic trends in the amplitudes of the harmonic oscillations are not distinguishable.

Another peculiarity of the heat transfer process revealed in the model simulations is the appearance of the non-zero double-frequency harmonic ($k = 2$) in the temperature variations and the difference between the mean surface temperature $\langle T_s \rangle$ and the mean temperatures in snow and firn deposits T_0 . Both effects are linked to the interaction of the surface temperature seasonal variations with those of the mass balance rate and are illustrated by Fig. 5b. In our case these variations do not differ much in phase (Table 1). Hence, a larger fraction of snow is accumulated in relatively warmer periods, and the mean snow temperatures are higher in the interior of the glacier than on its surface. Although the theoretical predictions show that the temperature excess and the

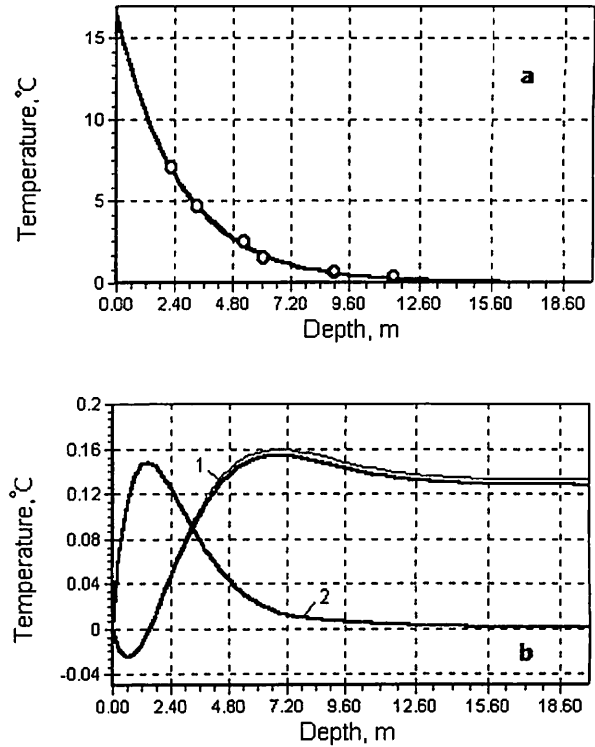


Fig. 5. (a) Comparison of the measured (dots) and simulated (solid line) amplitudes of the dominant first harmonic in the seasonal temperature variations plotted against depth. (b) The simulated mean temperature excess (curves 1) and the amplitude of the second harmonic (curve 2) of the seasonal temperature variations in snow-firn deposits versus depth. The thick and thin lines correspond to $N = 1$ and 2 in Eq. (4), respectively.

second harmonic are too small to be detectable in the particular experiments they are important for general understanding of the nature of the temperature conditions imposed on the glacier on annual basis. All our conclusions do not change if a more detailed expansion of the mass-balance fluctuations is used. For example, in case II for $N = 2$ in Eq. (4) the only but small difference can be found in estimates of T_0 (compare thick and thin curves 1 in Fig. 5b).

Due to the rapid increase in the mean snow temperature within the upper 3–4 meters of the near-surface snow deposits, the simulated signal of the first sensor has a linear trend in temperature variations $c_0 \approx 0.1^\circ\text{C yr}^{-1}$. Although this is within the accuracy limits of the data processing, the actual short-term rate of the climatic decrease in annual temperature on the ice-cap surface should be the difference between the above value and what is found in the readings of the first sensor in Table 2. Totally the estimate of the rate of the annual surface temperature decrease during the 1996–1998 period is about $-2.3^\circ\text{C yr}^{-1}$.

Special series of computational experiments are performed to test the sensitivity of the model simulations to the relative thermal conductivity of snow and firn $\Lambda(c)$, that is to the parameter a in Table 1. The values of a range from 0.4 to 0.6 for the $\pm 1^\circ\text{C}$ limits of

changes in the amplitude of the surface temperature variations to maintain the same agreement between the measured and simulated amplitudes of the seasonal temperature oscillations in snow-firn deposits as shown in Fig. 5a. This result confirms the previous estimates given by Salamatin *et al.* (2000).

5. Conclusion

A mathematical model is developed to simulate the seasonal temperature variations in the near-surface snow-firn stratum of an ice cap in cold snow accumulation areas at high altitudes. The model is applied to interpret the continuous temperature records obtained during the 1996-1998 period from the sensors non-uniformly moving in the 27-meter deep borehole BH-1 drilled in the center of the Gorshkov Ice Cap on the summit of Ushkovsky Volcano at Kamchatka. The amplitude of the surface-snow temperature oscillations and the short-term linear climatic trend are deduced. Thermal conductivity of the snow-firn deposits as a function of the porosity is constrained by the data of the field studies.

References

- Budd, W. F. (1969): The dynamics of ice masses. ANARE Sci. Rep.108.
- Hobbs, P. V. (1974): Ice Physics, Oxford, Clarendon Press.
- Kobayashi, D., Muravyev, Ya. D., Kodama, Y. and Shiraiwa, T. (1997): An outline of Russo-Japanese joint glacier research at Kamchatka, 1996. Bull. Glacier Res., **15**, 19-26.
- Matsuoka, K., Horikawa, S., Shiraiwa, T., Muravyev, Ya. D., Salamatin, A. N., Ovsyannikov, A. A., Maeno, H. and Ohi, M. (1999): Radio echo sounding at the summit ice cap of Ushkovsky Volcano, Kamchatka, In Cryospheric Studies in Kamchatka II, Institute of Low Temperature Science, Hokkaido University, 20-24.
- Muravyev, Ya. D. and Salamatin, A. N. (1989): Balans massy i tremodinamicheskii regim lednika v kratere Ushkovskogo vulkana [Mass balance and thermal regime of a crater glacier at Ushkovsky Volcano]. Vulkanologiya i Seismologiya, **3**, 85-92 (English Translation: Volcanology and Seismology, **11**(3), 411-423, 1990).
- Salamatin, A. N. (1991): Ice sheet modelling taking account of glacier ice compressibility. In Glaciers-Ocean-Atmosphere Interaction (Proceedings of Symposium held at St. Petersburg, 1990), IAHS Publication no. **208**, 183-192.
- Salamatin, A. N., Muravyev, Ya. D., Shiraiwa, T. and Matsuoka, K. (2000): Modelling dynamics of glaciers in volcanic craters. J. Glaciol., **46**(153), 177-187.
- Shiraiwa, T., Muravyev, Ya. D., Kameda, T., Nishio, F., Toyama, Y., Takahashi, A., Ovsyannikov, A. A., Salamatin, A. N. and Yamagata, K. (in press): Characteristics of a crater glacier at Ushkovsky volcano as revealed by the physical properties of its ice cores and borehole thermometry. J. Glaciol., in press.
- Shiraiwa, T., Nishio, F., Kameda, T., Takahashi, A., Toyama, Y., Muravyev, Ya. D., and Ovsyannikov, A. A. (1999a): Ice core drilling at Ushkovsky ice cap, Kamchatka, Russia. Seppyo, **61**(1), 25-40. (In Japanese with English abstract)
- Shiraiwa, T., Muravyev, Ya. D., Matsuoka, K., Salamatin, A. N., Horikawa, S., Ovsyannikov, A. A., Fujikawa, T. and Tanaka, N. (1999b): Geophysical and paleoclimatic implications of the Ushkovsky Ice Cap in Kamchatka. In Cryospheric Studies in Kamchatka II, Institute of Low Temperature Science, Hokkaido University, 8-19.
- Shiraiwa, T., Muravyev, Ya. D. and Yamaguchi, S. (1997): Stratigraphic features of firn as climate proxy signals at the summit ice cap of Ushkovsky Volcano, Kamchatka, Russia. Arc. Alp. Res., **29**(4), 414-421.
- Sturm, M., Holmgren, J., König, M. and Morris, K. (1997): The thermal conductivity of seasonal snow. J. Glaciol., **43**(143), 26-41.
- Vostretsov, R. N., Dmitriyev, D. N., Putikov, O. F., Blinov, K. V. and Mitin, S. V. (1984): Osnovnyye rezul'taty geofizicheskikh issledovaniy glubokikh skvazhin i ledyanogo kerna v Vostochnoy Antarktide [The main results of geophysical studies of deep boreholes and the ice core in East Antarctica]. Mater. Glyatsiol. Issled., **51**, 172-178. (In Russian)